A LEXICOGRAPHIC GOAL PROGRAMMING APPROACH FOR A BI-OBJECTIVE TRANSPORTATION PROBLEM

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ABSTRACT
In this paper, we have considered a Bi-Objective Transportation Problem, minimizing the total transportation cost and delivery time. A solution approach using D_1 – distances in the lexicographic goal programming is used to solve. The solution corresponding to the minimum D_1 – distances gives the best compromise solution. A numerical example is also given to illustrate the procedure.

Key Words: Compromise allocation; Lexicographic Goal Programming; ASM –Method; Bi-objective Transportation Problem.

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1. INTRODUCTION

The transportation problem is one of the oldest applications of Linear Programming Problem (LPP). The standard form of the transportation problem was first formulated along with the constructive method of solution by Hitchcock (1941). The transportation problem can be converted by to a standard Linear Programming Problem and can be solved by simplex method. In a classical transportation problem, a product is to be transported from $'m'$ sources to $'n'$ destinations. The availability of that product at $'m'$ sources to $'n'$ destinations are $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_n$ respectively. The penalty $'c_{ij}'$ that is the coefficients of the objective function can represent transportation cost, delivery time etc. This discussion was all about when the objective that is to be optimized was single but in many practical situations multiple penalty criteria may exist concurrently, which forced researchers to think and work for multi-objective transportation problem. Lee and Moore (1973) have studied the multi-objective optimization of the transportation problems. Iserman (1979) presented an algorithm for solving linear multi-objective transportation problems by which the set of all efficient solutions was enumerated. Ringuest and Rinks (1987) also proposed two approaches for obtaining the solution of linear multi- objective transportation problems. Bit et al. (1992) also developed a procedure for solving multi-criteria decision making transportation problems. Panda *et al.* (2005) have discussed the EOQ of multi-item inventory problems through nonlinear goal programming. Das *et al.* (1999) used fuzzy programming approach for solving multi-objective interval transportation problem. Waiel F. and El-Wahed (2001) studied multi-objective transportation problem under fuzziness. Ali *et al.* (2011a, 2011b) have also used the integer nonlinear goal programming approach in multi-objective reliability optimization and sample surveys problem.

In the present paper a Bi-Objective Transportation Problem is considered and the solution is obtained by using lexicographic goal programming Technique with "Minimum D_1 – distances". The individual objective optimal solution is obtained at first stage with help of ASM method (2012). LINGO software package is also used.

2. MULTI-OBJECTIVE TRANSPORTATION PROBLEM

The mathematical model of Multi Objective Transportation Problem (MOTP) is as follows:

$$\left.\begin{array}{lr}
Minimize\ F^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^k\ x_{ij} & (i) \\
Subject\ to & \\
\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1,2\ldots m & (ii) \\
\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1,2\ldots n & (iii) \\
\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (balanced\ condition) & (iv) \\
x_{ij} \geq 0, i = 1,2\ldots m;\ \ j = 1,2\ldots n. & (v)
\end{array}\right\} \qquad (1)$$

Where $F^k(x) = \{F^1(x), F^2(x) \ldots F^K(x)\}$ is a vector of K objective functions and the superscript on both $F^k(x)$ and C_{ij}^k are used to indicate the number of objective functions $k = 1,2\ldots K$. The solution procedure described below is indented for only two objectives (i.e. transportation cost and delivery time). The Bi-Objective

Transportation Problem (BOTP) is obtained on putting $K = 2$ in equation (1) as

$$\left.\begin{array}{ll}
Min\ F^1(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^1\ x_{ij}\,(Cost\ Minmizing\ Objective) & (i)\\
and & \\
Min\ F^2(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^2\ x_{ij}\,(Time\ Minmizing\ Objective) & (ii)\\
Subject\ to & \\
\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2\ldots m & (iii)\\
\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2\ldots n & (iv)\\
\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad (balanced\ condition) & (v)\\
x_{ij} \geq 0, \quad i = 1,2\ldots m; \quad j = 1,2\ldots n. & (vi)
\end{array}\right\} \quad (2)$$

3. $D_1 -$ DISTANCE METHOD OF LEXICOGRAPHIC GOAL PROGRAMMING

In the problem (2), let us first consider the total transportation cost is more important than the total delivery time. Then we solve the problem (2) by minimizing (*i*) subject to (*iii*) to (*vi*) (*i.e.* we neglect the objective (*ii*)). Let the minimum of the (BOTP) (2), while neglecting the $F^2(x)$ (i.e. second objective) be $F^1(x)^*$. Next we solve the following (BOTP):

$$\left.\begin{array}{l}
Minimize\ F^2(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^2\ x_{ij} + \delta_1\\
Subject\ to\ \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^1\ x_{ij} - \delta_1 \leq F^1(x)^*\\
\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,2\ldots m\\
\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,2\ldots n\\
\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j\ \ (balanced\ condition)\\
x_{ij} \geq 0, \quad i = 1,2\ldots m;\ j = 1,2\ldots n.
\end{array}\right\} \quad (3)$$

where δ_1 is the deviational variable

By solving the BOTP (3) let the optimum delivery time obtained be $F^2(x)^*$. The following lexicographic goal programming problem is then solved:

$$\left.\begin{array}{l}
Minimize = \delta_1 + \delta_2\\
Subject\ to\\
\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^1\ x_{ij} - \delta_1 \leq F^1(x)^*\\
\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^2\ x_{ij} - \delta_2 \leq F^2(x)^*\\
\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,2\ldots m\\
\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,2\ldots n\\
\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad (balanced\ condition)\\
x_{ij} \geq 0, \quad i = 1,2\ldots m;\ j = 1,2\ldots n\\
\delta_1,\ \delta_2 \geq 0.
\end{array}\right\} \quad (4)$$

Let the solution of the BOTP (4) obtained be $(x_{11}^{(1)},\ldots,x_{mn}^{(1)})$.

Next we assume that the total transportation delivery time is more important than the total transportation cost. Then we solve the BOTP (2) by considering the delivery time objective and neglecting the total transportation cost objective. Let the minimum so obtained be $F^2(x)^*$.

In the next step solve the following BOTP for optimum total transportation cost

$$\left.\begin{array}{l}
Minimize\ F^1(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^1\ x_{ij} + \delta_1\\
Subject\ to\ \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^2\ x_{ij} - \delta_1 \leq F^2(x)^*\\
\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,2\ldots m\\
\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,2\ldots n\\
\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j\,(balanced\ condition)\\
x_{ij} \geq 0, \quad i = 1,2\ldots m;\ j = 1,2\ldots n.
\end{array}\right\} \quad (5)$$

Let the minimum cost obtain be $F^1(x)^*$.

The following lexicographic goal programming problem is then solved:

$$\left.\begin{aligned} & Minimize = \delta_1 + \delta_2 \\ & Subject\ to \\ & \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^2\ x_{ij} - \delta_1 \le\ F^2(x)^* \\ & \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^1\ x_{ij} - \delta_2 \le F^1(x)^* \\ & \sum_{j=1}^{n} x_{ij} = a_i,\ \ i = 1,2\ldots m \\ & \sum_{i=1}^{m} x_{ij} = b_j,\ \ j = 1,2\ldots n \\ & \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (balanced\ condition) \\ & x_{ij} \ge 0, \quad i = 1,2\ldots m;\ \ j = 1,2\ldots n \\ & \delta_1,\ \delta_2 \ge 0. \end{aligned}\right\} \tag{6}$$

Let the solution of the BOTP (6) obtained be $(x_{11}^{(2)},\ldots,x_{mn}^{(2)})$.

In this way the priorities are given to the objectives one after the other and a set of solutions is obtained. Out of these solutions, an ideal solution is identified as follows:

$$x_{ij}^* = \left\{\min(x_{11}^{(1)}, x_{11}^{(2)}), \min(x_{22}^{(1)}, x_{22}^{(2)}),\ldots,\min(x_{mn}^{(1)}, x_{mn}^{(2)})\right\}$$
$$= \left\{x_{11}^*, x_{22}^*,\ldots, x_{mn}^*\right\}$$

The D_1-distances of different solutions from the ideal solution defined in (8) below are then calculated. The solution corresponding to the minimum D_1-distance gives the best compromise solution.

A general procedure with P objectives is the following. As explained above, we will obtain $P!$ (Factorial) different solutions by solving the $P!$ problems arising for $P!$ different priority structures.

Let $x_{ij}^{(r)} = \left\{x_{11}^{(r)}, x_{22}^{(r)},\ldots, x_{mn}^{(r)}\right\}, 1 \le r \le P!$ be the $P!$ number of solutions obtained by giving priorities to P objective functions. Let $(x_{11}^*, x_{22}^*,\ldots, x_{mn}^*)$ be the ideal solution. But in practice ideal solution can never be achieved. The solution, which is closest to the ideal solution, is acceptable as the best compromise solution, and the corresponding priority structure is identified as most appropriate priority structure in the planning context. To obtain the best compromise solution, following goal programming problem is to be solved.

$$\left.\begin{aligned} & \underset{1\le r\le\ P!}{Min}\ \ \sum_{i=1}^{m} \varepsilon_{ir} \\ & \text{subject to} \\ & x_{ij}^* - x_{ij}^{(r)} - \varepsilon_{ir} = 0 \\ & and\ \ \varepsilon_{ir} \ge 0,\ 1 \le r \le P\ ! \\ & x_{ij}^r \ge 0,\ \ i = 1,2\ldots m;\ \ j = 1,2\ldots n \end{aligned}\right\} \tag{7}$$

where ε_{ir} are the deviational variables.

Now,

$$(D_1)^r = \sum_{i=1}^{m}\sum_{j=1}^{n}\left|x_{ij}^* - x_{ij}^{(r)}\right| \tag{8}$$

is defined as the D_1-distance from the ideal solution $(x_{11}^*, x_{22}^*,\ldots, x_{mn}^*)$, to the $r-th$ solution $\left\{x_{11}^{(r)}, x_{22}^{(r)},\ldots, x_{mn}^{(r)}\right\}$, $1 \le r \le P!$

Therefore,

$$(D_1)_{opt} = \underset{1\le r\le P!}{Min}(D_1)^r = \underset{1\le r\le P!}{Min}\sum_{i=1}^{m}\left|x_{ij}^* - x_{ij}^{(r)}\right| \tag{9}$$

$$= \underset{1\le r\le P!}{Min}\sum_{i=1}^{m}\varepsilon_{ir} \tag{10}$$

Let the minimum be attained for $r = p$

Then

$\left\{x_{11}^{(p)}, x_{22}^{(p)},\ldots, x_{mn}^{(p)}\right\}$ is the best compromise solution of the problem.

3. NUMERICAL ILLUSTRATION

Consider a bi-objective transportation problem with three sources and three destinations as follows:

Data for time:

	D_1	D_2	D_3	Supply
S_1	16	19	12	14
S_2	22	13	19	16
S_3	14	28	8	12
Demand	10	15	17	42

Data for cost:

	D_1	D_2	D_3	Supply
S_1	9	14	12	14
S_2	16	10	14	16
S_3	8	20	6	12
Demand	10	15	17	42

$$\left.\begin{aligned}
&Min\, F^1 = 16x_{11} + 19x_{12} + 12x_{13} + 22x_{21} + 13x_{22} \\
&\qquad + 19x_{23} + 14x_{31} + 28x_{32} + 8x_{33} \\
&and \\
&Min\ \ F^2 = 9x_{11} + 14x_{12} + 12x_{13} + 16x_{21} + 10x_{22} \\
&\qquad + 14x_{23} + 8x_{31} + 20x_{32} + 6x_{33} \\
&subject\ \ to: \\
&\sum_{j=1}^{3} x_{1j} 14,\ \sum_{j=1}^{3} x_{2j} = 16,\ \sum_{j=1}^{3} x_{3j} = 14 \\
&\sum_{i=1}^{3} x_{i1} = 10,\ \sum_{i=1}^{3} x_{i2} = 15,\ \sum_{i=1}^{3} x_{i3} = 17 \\
&x_{ij} \ge 0,\quad i = 1,2,3;\quad j = 1,2,3.
\end{aligned}\right\}$$

Solution by using Lexicographic Goal Programming Approach:

Table 1

	D1	D2	D3	Supply
S1	16 [9]	19	12 [5]	14
S2	22 [1]	13 [1]	19	16
S3	14	28	8 [12]	12
Demand	10	15	17	42

The BOTP (2) is solved by giving priority to the delivery time objective using the values are given in Table 1. The optimal values obtained from (2) and (3) are $F^1(x)^* = 517$ and $F^2(x)^* = 375$. Next we solve the following NLPP corresponding to (4):

$$\left.\begin{aligned}
&Minimize \qquad \delta_1 + \delta_2 \\
&Subject\ \ to \\
&16x_{11} + 19x_{12} + 12x_{13} + 22x_{21} + 13x_{22} + 19x_{23} \\
&\qquad + 14x_{31} + 28x_{32} + 8x_{33} - \delta_1 \le 517 \\
&9x_{11} + 14x_{12} + 12x_{13} + 16x_{21} + 10x_{22} + 14x_{23} \\
&\qquad + 8x_{31} + 20x_{32} + 6x_{33} - \delta_2 \le 375 \\
&\sum_{j=1}^{3} x_{1j} = 14,\ \sum_{j=1}^{3} x_{2j} = 16,\ \sum_{j=1}^{3} x_{3j} = 12 \\
&\sum_{i=1}^{3} x_{i1} = 10,\ \sum_{i=1}^{3} x_{i2} = 15,\ \sum_{i=1}^{3} x_{i3} = 17 \\
&x_{ij} \ge 0,\quad i = 1,2,3;\quad j = 1,2,3.
\end{aligned}\right\} \qquad (11)$$

The solution to the BOTP (11) using software LINGO is

$x_{11}^* = 9.8,\ x_{13}^* = 4.20,\ x_{22}^* = 15,\ x_{23}^* = 0.80,\ x_{33}^* = 12,\ x_{12}^* = 0,\ x_{21}^* = 0.20,\ x_{31}^* = x_{32}^* = 0$

with $\delta_1 = 0.80\ \ and\ \ \delta_2 = 0$

Table 2

	D1	D2	D3	Supply
S1	9 [10]	14	12 [4]	14
S2	16	10 [15]	14 [1]	16
S3	8	20	6 [12]	12
Demand	10	15	17	42

In similar manner, NLPP (2) is solved by giving priority to the total transportation cost objective and values are given in Table 2. we obtain the values $F^2(x)^* = 374$ and $F^1(x)^* = 518$. Next solve the following NLPP corresponding to (6):

$$\left.\begin{aligned}
&Minimize \qquad \delta_1 + \delta_2 \\
&Subject\ to \\
&9x_{11} + 14x_{12} + 12x_{13} + 16x_{21} + 10x_{22} + 14x_{23} + 8x_{31} + 20x_{32} + 6x_{33} - \delta_2 \le 374 \\
&16x_{11} + 19x_{12} + 12x_{13} + 22x_{21} + 13x_{22} + 19x_{23} + 14x_{31} + 28x_{32} + 8x_{33} - \delta_1 \le 518 \\
&\sum_{j=1}^{3} x_{1j} = 14 \\
&\sum_{j=1}^{3} x_{2j} = 16 \\
&\sum_{j=1}^{3} x_{3j} = 12 \\
&\sum_{i=1}^{3} x_{i1} = 10 \\
&\sum_{i=1}^{3} x_{i2} = 15 \\
&\sum_{i=1}^{3} x_{i3} = 17 \\
&x_{ij} \ge 0,\ \ i = 1,2,3;\ \ j = 1,2,3.
\end{aligned}\right\}$$

(12)

The solution to the BOTP (12) using software LINGO 13.0 is

$$x_{11}^* = 10,\ x_{13}^* = 4,\ x_{22}^* = 15,\ x_{23}^* = 1,\ x_{33}^* = 12,\ x_{12}^* = x_{21}^* = x_{31}^* = x_{32}^* = 0$$

with $\delta_1 = 0\ \ and\ \ \delta_2 = 0$

Table 3: (Solutions)

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Run	Priorities	x_{11}^*	x_{13}^*	x_{21}^*	x_{22}^*	x_{23}^*	x_{33}^*
1	$F^1(x)^{*(1)}, F^2(x)$	**9.8**	4.20	0.20	**15**	**0.80**	**12**
2	$F^2(x)^{*(1)}, F^1(x)$	10	**4**	**0**	**15**	1	**12**
Ideal solution (d_i^*)		9.8	4	0	15	0.80	12

Table 4: The D_1 – distance from the ideal solutions

priority to	x_{11}^*	x_{13}^*	x_{21}^*	x_{22}^*	x_{23}^*	x_{33}^*	$(D_1)^r$
delivery time	0	0.20	0.20	0	0	0	**0.40**
T. P. Cost	0.20	0	0	0	0.20	0	**0.40**

In Table 4 the D_1-distances of all possible solutions from the ideal solution are calculated. From Table 4 it is clear that the minimum of the D_1-distances of the two priority structure solutions from the ideal solutions are equal to **0.40**. Therefore the tie occurs. Thus, we may choose any one of the two priority structures. But for our problem we choose second priority structure because it is providing exactly $(m+n-1)$ number of allocations.

6. CONCLUSION

The D_1-distances method of lexicographic goal programming is a powerful technique for solving Bi-objective optimization problem. In real life situations applications of D_1-distances method of lexicographic goal programming are sound in engineering design etc. in this paper, the problem of allocation in transpiration problem is considered as a Bi-objective optimization problem. A solution procedure is developed to solve the resulting mathematical programming problem by using D_1-distances. The solution which is corresponding to minimum-D_1 - distance is the best compromise solution. The solution described here is much simpler than complex analytical techniques described in literature.

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BIOGRAPHICAL NOTES

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